EXERCISE – III SUBJECTIVE QUESTIONS

1. Show that points (0, 7, 10), (–1, 6, 6) and (–4, 9, 6) form an isosceles right angled triangle.

2. Prove that the tetrahedron with vertices at the points (0, 0, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0) is a regular tetrahedron. Find also the co-ordinates of its centroid.

3. Find the coordinates of the point equidistant from the point (a, 0, 0), (0, b, 0), (0, 0, c) and (0, 0, 0,).

4. Find the ratio in which the line joining the points (3, 5, –7) and (–2, 1, 8) is divided by the y-z plane. Find also the point of intersection on the plane and the line.

5. What are the direction cosines of a line that passes through the points P(6, –7, –1) and Q(2, –3, 1) and is so directed that it makes an acute angle α with the positive direction of x-axis.

6. Find the angle between the lines whose direction cosines are given by $\ell + m + n = 0$ and $\ell^2 + m^2 = n^2$.

7. Show that the foot of the perpendicular from the origin to the join of A(–9, 4, 5) and B(11, 0, –1) is the mid point of AB.

8. P and Q are the points (–1, 2, 1) and (4, 3, 5). Find the projection of PQ on a line which makes angles of 120° and 135° with y and z axes respectively and an acute angle with x-axis.

9. Find the equation of the planes passing through points (1, 0, 0) and (0, 1, 0) and making an angle of 0.25 π radians with plane x + y – 3 = 0.

10. Find the angle between the plane passing through point (1, 1, 1), (1, –1, 1), (–7, –3, –5) & x-z plane.

11. Find the equation of the plane containing parallel lines $(x - 4) = \frac{3-y}{4} = \frac{z-2}{5}$ and $(x - 3) = \lambda (y + 2) = \mu z$.

12. Find the equation of image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane 3x – 3y + 10z = 26.

13. Find the distance between points of intersection of

(i) Lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ & $\frac{x-4}{5} = \frac{y-1}{2} = z$

(ii) Lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ & $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

14. Find the equation of the sphere described on the line (2, –1, 4) and (–2, 2, –2) as diameter. Also find the area of the circle in which the sphere is intersected by the plane 2x + y – z = 3.

15. Find the plane π passing through the points of intersection of the planes 2x + 3y – z + 1 = 0 and x + y – 2z + 3 = 0 and is perpendicular to the plane 3x – y – 2z = 4. Find the image of point (1, 1, 1) in plane π.

16. Find the equation of the straight line which passes through the point (2, –1, –1); is parallel to the plane 4x + y + z + 2 = 0 and is perpendicular to the line of intersection of the planes 2x + y = 0, x – y + z.

17. If the distance between point (α, 5α, 10α) from the point of intersection of the lines

$\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 4\hat{j} + 12\hat{k})$ and

plane $\vec{r} = (\hat{i} - \hat{j} + \hat{k})$ = 5 is 13 units.

Find the possible values of α.

18. The edges of a rectangular parallelepiped are a, b, c; show that the angles between the four diagonals are given by $\cos^{-1}\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}$.

19. Find the equation of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\pi/3$.

20. Find the equation of the projection of line $3x - y + 2z - 1 = 0$, $x + 2y - z - 2 = 0$ on the plane $3x + 2y + z = 0$.

21. Find the acute angle between the lines $\frac{x-1}{\ell} = \frac{y+1}{m} = \frac{z}{n}$ & $\frac{x+1}{m} = \frac{y-3}{n} = \frac{z-1}{\ell}$ where $\ell > m > n$ and ℓ, m, n are the roots of the cubic equation $x^3 + x^2 - 4x = 4$.

22. Let P(1, 3, 5) and Q(−2, 1, 4) be two points from which perpendiculars PM and QN are drawn to the x-z plane. Find the angle that the line MN makes with the plane $x + y + z = 5$.

23. If 2d be the shortest distance between the lines $\frac{y}{b} + \frac{z}{c} = 1$; $x = 0$ $\frac{x}{a} - \frac{z}{c} = 1$; $y = 0$ then prove that $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$.

24. Prove that the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{1}$ lies in the plane $3x + 4y + 6z + 7 = 0$. If the plane is rotated about the line till the plane passes through the origin then find the equation of the plane in the new position.